CSE-217: Theory of Computation

REGULAR LANGUAGES

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Computational Model

What is a computer?

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finite state machine *or* **finite automaton.**

Automata

Finite Automata

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`What can a computer do with such a small memory?`

Example - 1

Hopcroft, Motowani and Ullman: Figure 1.1

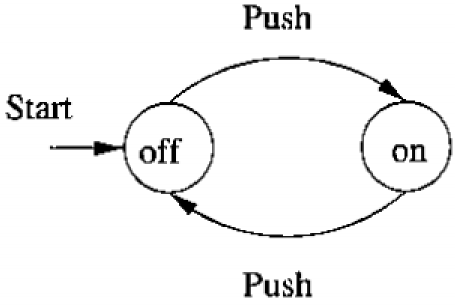

Figure: A finite automaton modeling an on/off switch

Example - 2

Michael Sipser: Figure 1.1

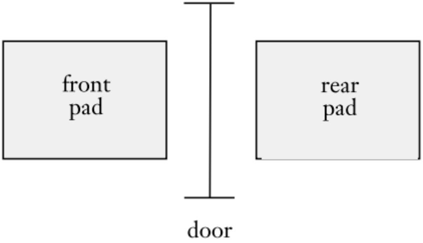

Figure: Top view of an automatic door

Example - 2

Michael Sipser: Figure 1.2

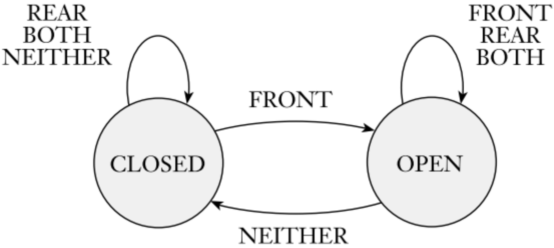

Figure: State diagram for an automatic door controller

Example - 3

Michael Sipser: Figure 1.4

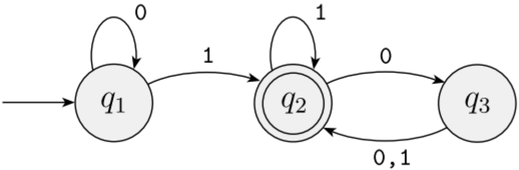

Figure: A finite automaton that has three states

Finite Automata

State diagram

- States
- Start State
- Accept State
- Transitions

Automata

Automata

- Finite Automata
- Infinite Automata

Automata

Automata

- Finite Automata
- Infinite Automata

Finite Automata

- Deterministic
- Non-deterministic

Finite Automata

Formal Definition

DEFINITION 1.5

A ***finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the ***states***,
2. Σ is a finite set called the ***alphabet***,
3. $\delta\colon Q \times \Sigma \longrightarrow Q$ is the ***transition function***,[1]
4. $q_0 \in Q$ is the ***start state***, and
5. $F \subseteq Q$ is the ***set of accept states***.[2]

Formal Definition

Language

- A is the set of all strings that machine M accepts.
- We say that A is the language of machine M.
- Write $L(M) = A$.
- We say that M recognizes A or that M accepts A.

Example - 3 continued

Michael Sipser: Figure 1.4

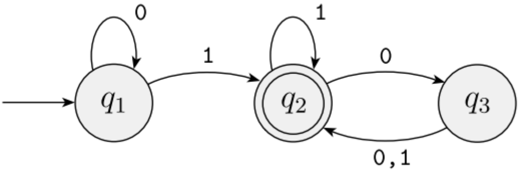

Figure: A finite automaton called M_1 that has three states

Example - 3 continued

We can describe M_1 formally by writing $M_1 = (Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3\}$,

2. $\Sigma = \{0,1\}$,

3. δ is described as

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2,

4. q_1 is the start state, and

5. $F = \{q_2\}$.

Example - 3 continued

$$A = \{w |\ w \text{ contains at least one } 1 \text{ and an even number of 0s follow the last } 1\}.$$

Then $L(M_1) = A$, or equivalently, M_1 recognizes A.

Example - 4

Michael Sipser: Figure 1.9

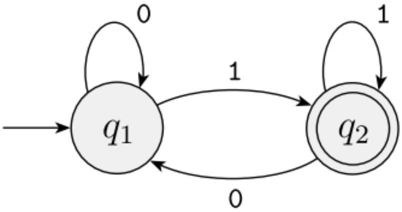

Figure: State diagram of the two-state finite automaton M_2

Example - 5

Michael Sipser: Figure 1.10

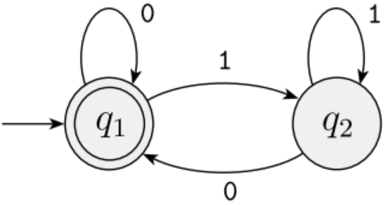

Figure: State diagram of the two-state finite automaton M_3

Example - 5

Michael Sipser: Figure 1.11

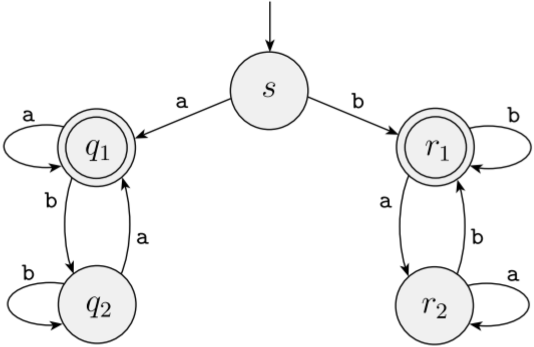

Figure: State diagram of the two-state finite automaton M_4

Example - 6

Michael Sipser: Figure 1.12

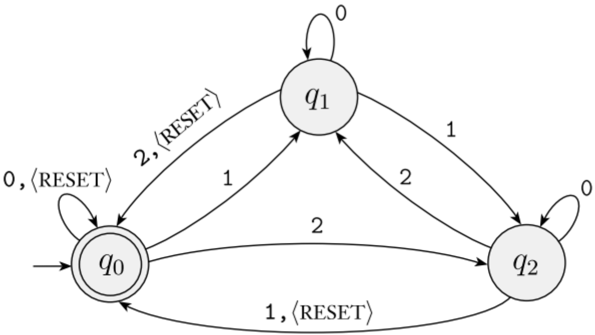

Figure: State diagram of the two-state finite automaton M_5

Definition of Computation

Formal definition of Computation

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton
- let $w = w_1 w_2 \dots w_n$ be a string
- each w_i is a member of the alphabet Σ.
- Then M accepts w if a sequence of states $r_0, r_1, \dots, r_n$ in Q exists with three conditions:

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1. $r_0 = q_0$,
2. $\delta(r_i, w_{i+1}) = r_{i+1}$ for $i = 0, \dots, n1$, and
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M **recognizes language** A if $A = \{w|M$ accepts $w\}$

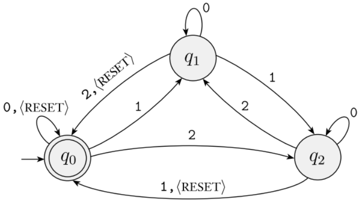
Formal definition of Computation

DEFINITION 1.16

A language is called a ***regular language*** if some finite automaton recognizes it.

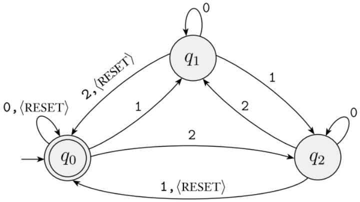
Example - 6

Michael Sipser: Figure 1.12



Example - 6

Michael Sipser: Figure 1.12



$$L(M_5) = \{w \mid \text{the sum of the symbols in } w \text{ is 0 modulo 3, except that } \langle\text{RESET}\rangle \text{ resets the count to 0}\}.$$

Thank You